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**195. Proposed by E. B. ESCOTT, University of Michigan.**

Find triangles whose sides are integers and one of whose angles is 60°.

### SOLUTION BY ARTEMAS MARTIN, Washington, D. C.

By trigonometry we have for *any* triangle the equation

$$x^2 - 2xy \cos \phi + y^2 = z^2, \tag{1}$$

in which $x$ and $y$ denote the sides including the angle $\phi$, and $z$ the side opposite that angle.

In this equation $\phi$ is supposed to be a given angle and $x$, $y$, $z$ wholly unknown and are to be determined in whole numbers.

When $\phi = 60°$, (1) reduces to

$$x^2 - xy + y^2 = z^2. \tag{2}$$

Assuming $z = x - \dfrac{qy}{p}$ we get from (2), after some reduction,

$$\frac{y}{x} = \frac{p^2 - 2pq}{p^2 - q^2};$$

therefore we may take

$$x = p^2 - q^2 \text{ and } y = p^2 - 2pq.$$

Substituting these values of $x$ and $y$ in the assumed value of $z$ we find

$$z = p^2 - pq + q^2.$$

Solving (2) for $y$ we get

$$y = \tfrac{1}{2}x \pm \sqrt{z^2 - \tfrac{3}{4}x^2}. \tag{3}$$

This result shows that $y$ has two values, and that there are two triangles for each value of $x$ and $z$.

Substutiting the values of $x$ and $y$ found above in (3) we get

$$\begin{aligned} y &= \tfrac{1}{2}(p^2 - q^2) \pm \tfrac{1}{2}(p^2 - 4pq + q^2), \\ &= p^2 - 2pq \text{ or } 2pq - q^2. \end{aligned}$$

It can be shown by a geometrical construction that $y$ has two values.

From what has been done it is obvious that the required triangles for each value of $p$ and $q$ are

$$p^2 - q^2, \quad p^2 - 2pq, \quad p^2 - pq + q^2;$$

and

$$p^2 - q^2, \quad 2pq - q^2, \quad p^2 - pq + q^2;$$

where $p$ can have any value, but $q$ is limited by the following restrictions:

1. $q$ must be less than $p$ and prime to it;
2. $q$ must not be greater than $\frac{1}{2}(p - 1)$ when $p$ is odd, and must be less than $\frac{1}{2}p$ when $p$ is even;
3. $p + q$ must not be divisible by 3, as in that case the sides of the triangle will be divisible by 3.

The following table contains the first seven pairs of such triangles.

| $p$ | $q$ | Sides | | |
|---|---|---|---|---|
| | | $p^2-q^2$ | $p^2-2pq$,<br>$2pq-q^2$ | $p^2-pq+q^2$ |
| 3 | 1 | 8 | 3 | 7 |
| 3 | 1 | 8 | 5 | 7 |
| 4 | 1 | 15 | 8 | 13 |
| 4 | 1 | 15 | 7 | 13 |
| 5 | 2 | 21 | 5 | 19 |
| 5 | 2 | 21 | 16 | 19 |
| 6 | 1 | 35 | 24 | 31 |
| 6 | 1 | 35 | 11 | 31 |
| 7 | 1 | 48 | 35 | 43 |
| 7 | 1 | 48 | 13 | 43 |
| 7 | 3 | 40 | 7 | 37 |
| 7 | 3 | 40 | 33 | 37 |
| 8 | 3 | 55 | 16 | 49 |
| 8 | 3 | 55 | 39 | 49 |

Also solved by THOMAS E. MASON, A. H. HOLMES, B. F. YANNEY, H. E. TREFETHEN, HORACE OLSON, N. A. DRAXTEN, E. B. ESCOTT, and C. E. GITHENS.

# MISCELLANEOUS QUESTIONS.

EDITED BY R. D. CARMICHAEL.

In the teaching of mathematics there are three fields of experience, now more or less separated, which it is desirable to have joined into one; namely, that of teachers in the better high schools, that of teachers in the smaller colleges and that of teachers in the larger and more powerful institutions. The problems in these fields are not as widely separated or as distinct as some have supposed. We need to find a means of funding the experience from all these three sources so that it may become the common possession of all who are at work in these fields.

It is our desire that this department of the MONTHLY shall become an effective means to this end. For this we need the coöperation of a large number of persons engaged in the teaching of mathematics. In this connection there are at least three ways in which our readers may render valuable service to the cause of mathematical education:

(*a*) Give us suggestions as to the plans along which this department should be developed.

(*b*) Propose specific questions on which it is desirable to know what is the general opinion or what is the experience of individual teachers.

(*c*) Respond promptly to the questions printed with a statement of your opinion or an account of your experience or both.

Already we have had a gratifying expression of interest in these matters. We have in hand now an excellent answer to question 5 on culture mathematics and an excellent answer to question 6 on vocational mathematics. Let us have further replies to these and to other questions; let us also have other useful questions proposed. If the interest invoked and the amount of good matter received requires it, we can enlarge this department in later issues so as to meet the demand made upon it.